LIATE $\int t\,dv = tv - \int v\,dt$

1) a) $\int x^3 e^{x^2} dx = \int x\,x^2 e^{x^2} dx = \frac{1}{2}\int u e^u du =$

$u = x^2$
$du = 2x\,dx$

$\begin{cases} t = u \Rightarrow dt = du \\ v = e^u \quad dv = e^u\,du \end{cases}$

$= \frac{1}{2}\left[ue^u - \int e^u du\right] = \frac{1}{2}\left[ue^u - e^u\right] + C$

$= \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C = \frac{1}{2}e^{x^2}\left[x^2 - 1\right] + C$ //

b) $\int_0^{+\infty} x e^{-x^2} dx = \lim_{t \to +\infty} \int_0^t x e^{-x^2} dx$

$u = -x^2$
$du = -2x\,dx$

$= \lim_{t\to+\infty} \int \frac{-1}{2} e^u du = \lim_{t\to+\infty} \left(\frac{-1}{2}e^u\right)\Big|$

$= \lim_{t\to+\infty} \left(-\frac{1}{2}e^{-x^2}\right)\Big|_0^t = \lim_{t\to+\infty}\left(\cancel{-\frac{1}{2e^{t^2}}}^{\to 0} + \frac{1}{2e^0}\right)$

$= \frac{1}{2}$ //

c) $\int_0^5 \frac{1}{x-1} dx = \lim_{t\to 1^-} \int_0^t \frac{1}{x-1} dx + \lim_{t \to 1^+} \int_t^5 \frac{1}{x-1} dx$

$= \lim_{t\to 1^-} \ln|x-1|\Big|_0^t + \lim_{t\to1^+} \ln|x-1|\Big|_t^5$

$= \lim_{t\to 1^-}\left[\underbrace{\ln|t-1|}_{-\infty} - \ln|0-1|\right] + \lim_{t\to1^+}\left[\ln|5-1| - \underbrace{\ln|t-1|}_{+\infty}\right]$

$\Rightarrow$ diverge!

d) $\int \frac{x^4+2}{x^3-4x^2+5x-2} dx$

1	1	-4	5	-2	
1	1	-3	2	0	
2	1	-2	0		
	1	0			

$$\begin{array}{l|l} x^4+2 & x^3-4x^2+5x-2 \\ \hline -x^4+4x^3-5x^2+2x & x+4 \\ \quad 4x^3-5x^2+2x+2 & \\ -4x^3+16x^2-20x+8 & \\ \hline \quad 11x^2-18x+10 & \end{array}$$

$$\int\left[x+4+\frac{11x^2-18x+10}{x^3-4x^2+5x-2}\right]dx$$

$$\frac{11x^2-18x+10}{(x-1)^2(x-2)}=\frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$=\frac{A(x-1)^2+B(x-2)(x-1)+C(x-2)}{(x-2)(x-1)^2}$$

$$=\frac{A(x^2-2x+1)+B(x^2-3x+2)+Cx-2C}{(x-2)(x-1)^2}$$

$$=\frac{Ax^2-2Ax+A+Bx^2-3Bx+2B+Cx-2C}{(x-2)(x-1)^2}$$

$$=\frac{(A+B)x^2+(-2A-3B+C)x+(A+2B-2C)}{(x-2)(x-1)^2}$$

$$\begin{cases} A+B=11 \\ -2A-3B+C=-18 \\ A+2B-2C=10 \end{cases} \qquad \begin{cases} A+B=11 \\ -2A-3B+C=-18 \\ -3A-4B=-26 \end{cases} \qquad \begin{cases} \cancel{3A+3B=33} \\ A+B=11 \\ -3A-4B=-26 \end{cases}$$

$$-B=7$$

$$B=-7$$

$$A=11+7=+18$$

$$C=-18+2A+3B$$

$$C=-18+36-21$$

$$C=-3$$

$$\int\left[x+4+\frac{18}{x-2}-\frac{7}{x-1}-\frac{3}{(x-1)^2}\right]dx=$$

$$=\frac{x^2}{2}+4x+18\ln|x-2|-7\ln|x-1|+\frac{3}{x-1}+C$$

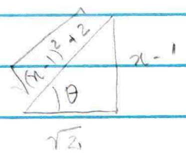
e) $\displaystyle\int \frac{x}{\sqrt{3-2x+x^2}}\,dx = \int \frac{x}{\sqrt{(x-1)^2+2}}\,dx$

$x-1 = \sqrt{2}\,tg\theta$

$x = \sqrt{2}\,tg\,\theta + 1$

$dx = \sqrt{2}\,sec^2\theta\,d\theta$

$tg\theta = \frac{x-1}{\sqrt{2}}$



$sec\theta = \frac{\sqrt{(x-1)^2+2}}{\sqrt{2}}$

$u = cos\theta$

$du = -sen\theta\,d\theta$

$$= \int \frac{(\sqrt{2}\,tg\theta + 1)\,\cancel{\sqrt{2}}\,sec^{\cancel{2}}\theta}{\cancel{\sqrt{2}\,sec\theta}}\,d\theta$$

$$= \int (\sqrt{2}\,tg\theta\,sec\theta + sec\theta)\,d\theta$$

$$= \int \left(\sqrt{2}\,\frac{sen\theta}{cos^2\theta} + sec\theta\right)d\theta$$

$$= \int \frac{\sqrt{2}}{u^2}\,du + ln|sec\theta + tg\theta| + C$$

$$= \frac{\sqrt{2}}{u} + ln|sec\theta + tg\theta| + C$$

$$= \frac{\sqrt{2}}{cos\theta} + ln|sec\theta + tg\theta| + C$$

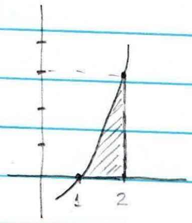
$$= \sqrt{(x-1)^2+2} + ln\left|\frac{\sqrt{(x-1)^2+2}}{\sqrt{2}} + \frac{x-1}{\sqrt{2}}\right| + C$$

f) $\displaystyle\int_{\pi/2}^{3\pi/4} sen^5x\,cos^3x\,dx = \int_{\pi/2}^{3\pi/4} sen^5x\,(1-sen^2x)\,cos\,x\,dx$

$$= \int_{\pi/2}^{3\pi/4} (sen^5x\,cos\,x - sen^7x\,cos\,x)\,dx$$

$$\left(\frac{sen^6x}{6} - \frac{sen^8x}{8}\right)\Bigg|_{\pi/2}^{3\pi/4} = \left(\frac{\sqrt{3}}{2}\right)^6\cdot\frac{1}{6} - \left(\frac{\sqrt{3}}{2}\right)^8\cdot\frac{1}{8} - \frac{1}{6} + \frac{1}{8} =$$

3) $f(x) = x^2 - 1$, eixo x , intervalo $[1, 2]$



a) $A = \int_1^2 (x^2-1)\,dx = \left.\frac{x^3}{3} - x\right|_1^2$

$= \frac{8}{3} - 2 - \frac{1}{3} + 1 = \frac{7}{3} - 1 = \frac{4}{3}$ u.a.

b) $V = \pi\int_1^2 (x^2-1)^2\,dx = \pi\int_1^2 (x^4 - 2x^2 + 1)\,dx = \pi\left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_1^2$

$= \pi\left[\frac{32}{5} - \frac{16}{3} + 2 - \frac{1}{5} + \frac{2}{3} - 1\right] = \pi\left[\frac{31}{5} - \frac{14}{3} + 1\right]$

$= \pi\left[\frac{93 - 70 + 15}{15}\right] = \frac{38\pi}{15}$ u.v.

$y = x^2 - 1 \Rightarrow x = \sqrt{y+1}$

c) $V = \pi\int_0^3 \left[2^2 - (\sqrt{y+1})^2\right]dy = \pi\int_0^3 \left(4 - (y+1)\right)dy$

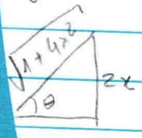
$= \pi\int_0^3 (3-y)\,dy = \pi\left[3y - \frac{y^2}{2}\right]_0^3 = \pi\left[9 - \frac{9}{2}\right] = \frac{9\pi}{2}$ u.v

d) $c = \int_1^2 \sqrt{1+4x^2}\,dx = \frac{1}{2}\int \sec\theta \cdot \sec^2\theta\,d\theta = \frac{1}{2}\int \sec^3\theta\,d\theta$

$x = \frac{1}{2}\,tg\,\theta$ $\qquad = \frac{1}{2}\left[\frac{1}{2}\,tg\,\theta \sec\theta + \frac{1}{2}\ln|\sec\theta + tg\,\theta|\right]$

$dx = \frac{1}{2}\sec^2\theta$ $\qquad = \frac{1}{2}\left[x\sqrt{1+4x^2} + \frac{1}{2}\ln\left|\sqrt{1+4x^2} + 2x\right|\right]_1^2$

$tg\,\theta = 2x$ $\qquad = \frac{1}{2}\left[2\sqrt{1+16} + \frac{1}{2}\ln\left|\sqrt{1+16} + 4\right| - \sqrt{1+4} - \frac{1}{2}\ln\left|\sqrt{5} + 2\right|\right]$



$= \sqrt{17} + \frac{1}{4}\ln\left|\sqrt{17} + 4\right| - \frac{\sqrt{5}}{2} - \frac{1}{4}\ln\left|\sqrt{5} + 2\right|$

$\sec\theta = \sqrt{1+4x^2}$